

Extended quintessence and its late-time domination

Takeshi Chiba

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

(Received 18 April 2001; published 19 October 2001)

Various astronomical observations point towards evidence of dark energy. One of the most mysterious problems is the coincidence problem: why dark energy has become dominant only recently. We present a scenario based on extended quintessence models to explain the late-time domination of dark energy without a severe fine-tuning of the initial conditions and model parameters.

DOI: 10.1103/PhysRevD.64.103503

PACS number(s): 98.80.Es, 98.80.Cq

I. INTRODUCTION

The evidence for dark energy now seems compelling [1]. Dark energy may be the cosmological constant or quintessence (dynamical lambda). In any case, it is very mysterious that dark energy appears to have become dominant only recently, although it should be negligible for most of the history of the universe for the success of the standard big-bang nucleosynthesis and structure formation: we live in a very special time when the dark energy density is comparable to matter density. The coincidence (or “why now”) problem [2] is the enigma in modern cosmology.

In order to “solve” the problem dynamically, the dark energy density should scale in the same way as the radiation density during the radiation dominated epoch; otherwise it is nothing but introducing a fine tuning to account for the coincidence from the very beginning, and it is no surprise that there is some epoch when the two energy components coincide (however, see [3]). On the other hand, however, during matter-dominated epoch, dark energy should not track matter; otherwise dark energy cannot dominate.

Several such dynamical approaches to solving the coincidence problem have been attempted that utilize noncanonical kinetic terms [4,5] (see also [6]) or the explicit coupling between quintessence and dark matter [7,8]. In this paper, we shall provide yet another mechanism based on extended quintessence models [9,10].

II. EXTENDED QUINTESSENTIAL APPROACH TO THE COINCIDENCE PROBLEM

Basic idea. We consider the cosmological dynamics described by the action [9–11]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \mathcal{R} - F(\phi) \mathcal{R} - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + S_m. \quad (1)$$

Here $\kappa^2 \equiv 8\pi G_{\text{bare}}$ is the bare gravitational constant and S_m denotes the action of matter (radiation and nonrelativistic particle). In our conventions, $\xi = 1/6$ corresponds to the conformal coupling.

The basic idea is to utilize the dynamics during inflation. Several quintessence models require the fine tuning of the initial conditions to account for the late-time domination of dark energy. The nonminimal coupling between the field and

the Ricci scalar provides a natural mechanism to tune the field itself to a value which is required to achieve the cosmic acceleration today.

Quintessence axion. As an example, we consider a model based on pseudo-Nambu-Goldstone bosons with the potential of the form [12]

$$V(\phi) = M^4 [\cos(\kappa\phi) + 1], \quad (2)$$

where M is a mass parameter. The smallness of M may be related to electroweak instanton effects or to a neutrino mass [12]. An attractive feature about this model is that the smallness of M is technically natural and is protected from various corrections by symmetry: When the small mass is set to zero, it cannot be generated in any order of perturbation theory. On the other hand, the weakness is that the dynamics is significantly dependent on the initial condition: there does not exist a tracker solution [13] in this model, and ϕ must be tuned near the top of the potential $\phi \sim 0$ in order to account for the late-time domination.

A nonminimal coupling can alleviate the situation. To demonstrate our point, we take the following functional form for $F(\phi)$ for simplicity:

$$F(\phi) = \frac{1}{2} \xi \phi^2, \quad (3)$$

where ξ is a dimensionless parameter and we assume $\xi > 0$ henceforth. The equations of motion in a FRW universe model are

$$\ddot{\phi} + 3H\dot{\phi} + V' + \xi\phi(\dot{H} + 2H^2) = 0, \quad (4)$$

$$3H^2 = \kappa^2 \left(\rho_B + \frac{1}{2} \dot{\phi}^2 + V + 3\xi H\phi(H\phi + 2\dot{\phi}) \right) \\ =: \kappa^2 (\rho_B + \rho_\phi), \quad (5)$$

where $V' = dV/d\phi$ and ρ_B denotes the background (radiation/matter) energy density.

During inflation which is caused by other fields, the field ϕ acquires an effective (time-dependent) mass due to the nonminimal coupling $m_{\text{eff}}^2 \approx \xi \mathcal{R}$ and is dynamically tuned

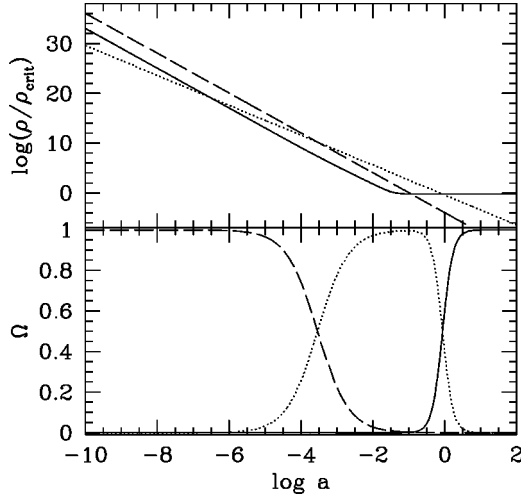


FIG. 1. The evolution of energy density for extended quintessence axion. The solid line is the energy density of dark energy, the dashed line is that of radiation, and the dotted line is that of matter. The lower panel shows the evolution of density parameters.

toward $\phi=0$ (if $\xi>0$)¹ which would subsequently correspond to the maximum of the potential induced by electroweak instanton effects or by a neutrino mass [12].² The evolution of ϕ during inflation is well approximated as $\phi(N)=\phi(N=0)\exp(-4\xi N)$ for $\xi\leq 1$, where N is the e-folding number after the initial time.

In Fig. 1 we show an example of the numerical results of the time evolution of the energy densities. The scale factor a is normalized to unity today. In this example, we take $\kappa\phi=0.1$ and $\dot{\phi}=0$ at $z=10^{12}$ and $\xi=0.1$ and arrange M to fix the density parameter of matter $\Omega_M=\kappa^2\rho_M/3H^2=0.4$ today. Interestingly, ρ_ϕ tracks the radiation energy density ρ_R during the radiation-dominated epoch (RD) unlike minimal quintessence axion (see below for the details). In the future ϕ will oscillate around a local minimum of the effective potential $V_{\text{eff}}(\phi)=V(\phi)+F(\phi)\mathcal{R}$ similar to the minimal quintessence axion. After the oscillations are damped, Ω_ϕ decreases suddenly and settles down to a small but nonvanishing value, which is estimated $\approx \xi\pi^2$ for $\xi\leq 1$. We note that the present Brans-Dicke parameter, $\omega_{BD}=(1-2\kappa^2 F)/4\kappa^2 F'^2$, is ≈ 16000 which is much larger than the current limit $\omega_{BD}>3500$ [15]. Since $\kappa\phi$ remains small, $\kappa\phi\ll 1$, large ω_{BD} is attainable even with not so small ξ . The situation is different from tracker models where generically $\kappa\phi\approx 1$ today and thus ξ is constrained to be very small, $\xi\lesssim 10^{-3}$ [9]. We also note that extended quintessence models with positive ξ can lead to a decrease in the primordial ^4He abundance because of the decrease of the gravitational constant in the past [16].³ However, the effect is found to be very small since $\xi\kappa\phi^2$ is small.

¹We thank Kazuya Koyama for pointing this out.

²The use of the nonminimal coupling for the restoration of symmetry in the inflationary stage was discussed in [14].

³We thank Kazunori Kohri for useful correspondence on this point.

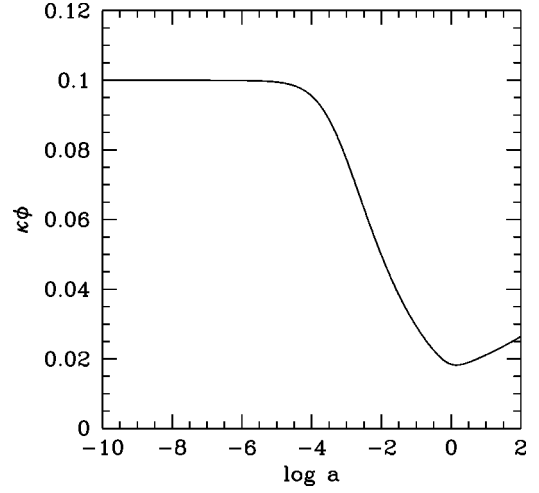


FIG. 2. The evolution of the scalar field for extended quintessence axion.

Tracking without tracking. Here we point out a novel tracking behavior during radiation-dominated epoch (RD) in extended quintessence which is independent of the details of the potential. Consider the situation where the energy density of the scalar field defined usually for the minimal coupling case is much smaller than the energy density of radiation:

$$\frac{1}{2}\dot{\phi}^2 + V \ll \rho_R, \quad (6)$$

and we assume that the curvature of the potential $\sqrt{V''}$ is smaller than the Hubble parameter H during RD, which is usually the case for most of the quintessence models. Then the motion of the scalar field is almost frozen because its energy density is dominated by the H^2 term in Eq. (5), even in the R-boost regime [11], which holds if at least one non-relativistic species is present during the RD. For $\xi\neq 0$, the energy density of the scalar field properly derived from the scalar field action $S_\phi=\int\sqrt{-g}[-(\nabla\phi)^2/2-V(\phi)-F(\phi)\mathcal{R}]$ becomes [17]⁴

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi)+6H(\dot{F}+HF)\approx 6H^2F. \quad (7)$$

Since $F(\phi)$ is constant, this implies that the ratio of ρ_ϕ to ρ_R is *constant* during RD although ϕ itself is *not* evolving. We shall call this behavior as “tracking without tracking.” As the universe becomes matter dominated the source term appears in the equation of motion of the scalar field, and the scalar field begins to move.

In fact, in the above example, such a behavior of ϕ is found. In Fig. 2 we plot the time evolution of ϕ . ϕ remains almost constant until the radiation-matter equality. The initial

⁴The corresponding expression for the pressure is $p_\phi=\dot{\phi}^2/2-V-2\ddot{F}-4H\dot{F}-2F(2\dot{H}+3H^2)$. The conservation of the energy momentum tensor of ϕ , $\dot{\rho}_\phi+3H(\rho_\phi+p_\phi)=0$, reduces to the Klein-Gordon equation, $\delta S_\phi/\delta\phi=0$.

decrease of ϕ after the radiation-matter equality (around $z \simeq 10^4$) is due to the curvature term induced by the nonminimal coupling. This is another dynamical tuning toward $\phi = 0$ during the matter-dominated epoch in addition to the inflationary epoch.

We should emphasize that this behavior is independent of the details of the shape of a potential. The only constraint is that the energy density usually defined for $\xi=0$ is much smaller than the radiation energy density and that the curvature of the potential is smaller than the Hubble parameter during RD. Moreover, the chosen initial value of ϕ is not critical and the described phenomenology remains valid for other smaller initial values of ϕ different from zero, although M should be fixed accordingly.⁵ However, it is also to be noted that this “tracking” is not a dynamical attractor (or tracker [13]). In the extended quintessence axion, for example, the dynamics during RD is dependent on the initial conditions like the minimal quintessence axion because the effect of the potential term in the equation of motion of the scalar field is negligible during RD. Tracking without tracking is a rather kinematical tracker inherent in a wide class of extended quintessence.

Exponential quintessence. We also present another model with an exponential potential, $V \propto \exp(-\lambda \kappa \phi)$. Minimal quintessence models involving exponential potentials have been extensively studied [18]. It has been shown that there exists the attractor solution which depends only on λ , but the solution tracks the matter/radiation energy density exactly in the same way and hence cannot account for the late-time domination of dark energy.

The situation can be different in extended quintessence. A slight modification of $F(\phi)$ enables us to construct a model based on an exponential potential which can account for the late-time domination of dark energy without the fine-tuning of parameters. For example, consider

$$F(\phi) = \frac{1}{2} \xi (\phi - v)^2, \quad (8)$$

$$V(\phi) = \kappa^{-4} \exp(-\lambda \kappa \phi). \quad (9)$$

The role of the shift parameter v is to reduce the energy scale of $V(\phi)$ by shifting ϕ without introducing small parameters

⁵However, as long as $\kappa \phi$ is close to zero, M takes almost the same value: For example, M decreases only 0.2% if one changes the initial value of $\kappa \phi$ from 0.2 to 0.

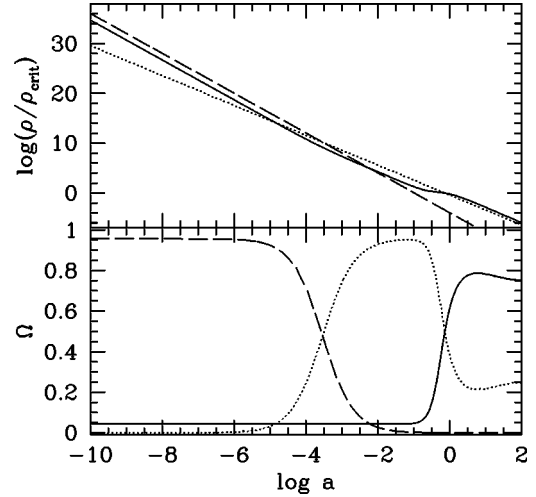


FIG. 3. The evolution of energy density for extended quintessence with an exponential potential.

in $V(\phi)$.⁶ In Fig. 3 we show the evolution of the energy density of each component. In this example, we take $\xi = 0.001, v = 144\kappa^{-1}, \lambda = 2$. We note that the present Brans-Dicke parameter is $\omega_{BD} \simeq 7030$.

III. SUMMARY

In this paper we have presented a scenario based on extended quintessence models to explain the late-time domination of dark energy without severe fine tuning of initial conditions and model parameters. If the scalar field is nonminimally coupled to the curvature, inflationary dynamics can provide the natural initial conditions required for the late-time domination of dark energy. Extended quintessence can subsequently track the radiation energy during the radiation dominated epoch even if minimal quintessence does not. This is due to a novel tracking behavior (tracking without tracking) which is inherent in a wide class of extended quintessence models. Further dynamical tuning occurs during matter-dominated epoch. Two models have been presented; one is based on the pseudo-Nambu-Goldstone bosons, which is attractive because the small mass is technically natural, the other is based on an exponential potential which only uses the parameters in the potential roughly of $O(1)$ in the Planck units. It will not be difficult to give other examples.

To conclude, extending quintessence to include nonminimal coupling to gravity greatly extends the range of viable models for quintessence.

ACKNOWLEDGMENTS

This work was supported in part by a Grant-in-Aid for Scientific Research (No. 13740154) from the Japan Society for the Promotion of Science.

⁶Introducing v may only paraphrase the smallness of the present energy scale of $V(\phi)$. But an interesting point may be that v is at most only $O(100)$ in $\kappa = 1$ unit (bare Planck units) [19].

- [1] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999); P. de Bernardis *et al.*, *Nature (London)* **404**, 955 (2000); A.H. Jaffe *et al.*, *Phys. Rev. Lett.* **86**, 3475 (2001).
- [2] M.S. Turner, *Phys. Scr.* **T85**, 210 (2000).
- [3] S. Dodelson, M. Kaplinghat, and E. Stewart, *Phys. Rev. Lett.* **85**, 5276 (2000).
- [4] V.A. Rubakov, *Phys. Rev. D* **61**, 061501 (2000); A. Hebecker and C. Wetterich, *Phys. Rev. Lett.* **85**, 3339 (2000).
- [5] C. Armendariz-Picon, V. Mukhanov, and P.J. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000).
- [6] T. Chiba, T. Okabe, and M. Yamaguchi, *Phys. Rev. D* **62**, 023511 (2000).
- [7] L. Amendola, *Mon. Not. R. Astron. Soc.* **312**, 521 (2000); *Phys. Rev. D* **62**, 043511 (2000).
- [8] L. Amendola and D. Tocchini-Valentini, *Phys. Rev. D* **64**, 043509 (2001).
- [9] T. Chiba, *Phys. Rev. D* **60**, 083508 (1999).
- [10] J.-P. Uzan, *Phys. Rev. D* **59**, 123510 (1999); L. Amendola, *ibid.* **60**, 043501 (1999); F. Perrotta, C. Baccigalupi, and S. Matarrese, *ibid.* **61**, 023507 (2000); O. Bertolami and P.J. Martins, *ibid.* **61**, 064007 (2000); B. Boisseau, G. Esposito-Farèse, D. Polarski, and A.A. Starobinsky, *Phys. Rev. Lett.* **85**, 2236 (2000); G. Esposito-Farèse and D. Polarski, *Phys. Rev. D* **63**, 063504 (2001).
- [11] C. Baccigalupi, S. Matarrese, and F. Perrotta, *Phys. Rev. D* **62**, 123510 (2000).
- [12] J.A. Frieman, C.T. Hill, and R. Watkins, *Phys. Rev. D* **46**, 1226 (1992); K. Choi, *ibid.* **62**, 043509 (2000); J.E. Kim, *J. High Energy Phys.* **06**, 016 (2000); Y. Nomura, T. Watari, and T. Yanagida, *Phys. Lett. B* **484**, 103 (2000).
- [13] P.J. Steinhardt, L. Wang, and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999).
- [14] J. Yokoyama, *Phys. Lett. B* **212**, 273 (1988).
- [15] C.M. Will, gr-qc/0103036.
- [16] X. Chen, R.J. Scherrer, and G. Steigman, *Phys. Rev. D* **63**, 123504 (2001).
- [17] A. Riazuelo and J.-P. Uzan, *Phys. Rev. D* **62**, 083506 (2000).
- [18] P.G. Ferreira and M. Joyce, *Phys. Rev. Lett.* **79**, 4740 (1997); E. Copeland, A.R. Liddle, and D. Wands, *Phys. Rev. D* **57**, 4686 (1998).
- [19] A. Albrecht and C. Skordis, *Phys. Rev. Lett.* **84**, 2076 (2000).